# Exhibit 26

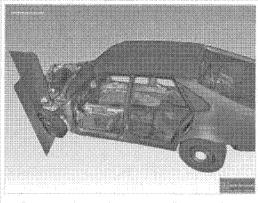
## Finite element analysis

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Finite element analysis (FEA) is a computer simulation technique used in engineering analysis. It uses a numerical technique called the finite element method (FEM). There are many finite element software packages, both free and proprietary. Development of the finite element method in structural mechanics is usually based on an energy principle such as the virtual work principle or the minimum total potential energy principle.

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Visualization of how a car deforms in an asymmetrical crash using finite element analysis.

## History

The finite element analysis from the mathematical side was first developed in 1943 by Richard Courant, who used the Ritz method of numerical analysis and minimization of variational calculus to obtain approximate solutions to vibration systems. From the engineering side, the finite element analysis originated as the displacement method of the matrix structural analysis, which emerged over the course of several decades mainly in British aerospace research as a variant suitable for computers<sup>[1]</sup>. By late 1950s, the key concepts of stiffness matrix and element assembly existed essentially in the form used today<sup>[2]</sup> and NASA issued request for proposals for the development of the finite element software NASTRAN in 1965.

## **Application**

In its applications, the object or system is represented by a geometrically similar model consisting of multiple, linked, simplified representations of discrete regions—i.e., finite elements on an unstructured grid. Equations of equilibrium, in conjunction with applicable physical considerations such as compatibility and constitutive relations, are applied to each element, and a system of simultaneous equations is constructed. The system of equations is solved for unknown values using the techniques of linear algebra or non-linear numerical schemes, as appropriate. While being an approximate method, the accuracy of the FEA method can be improved by refining the mesh in the model using more elements and nodes.

A common use of FEA is for the determination of stresses and displacements in mechanical objects and systems. However, it is also routinely used in the analysis of many other types of problems, including those in heat transfer, solid state diffusion and reactions with moving boundaries, fluid dynamics, and electromagnetism. FEA is able to handle complex systems that defy closed-form analytical solutions.

## Literature review of finite element analysis

Computer-aided engineering (CAE) is the application of computer software in engineering to evaluate components and assemblies. It encompasses simulation, validation, and optimization of products and manufacturing tools. The primary application of CAE, used in civil, mechanical, aerospace, and electronic engineering, takes the form of FEA alongside computer-aided design (CAD).

#### Finite element analysis

In general, there are three phases in any computer-aided engineering task:

- Pre-processing defining the finite element model and environmental factors to be applied to it
- Analysis solver solution of finite element model
- Post-processing of results using visualization tools

#### Pre-processing

The first step in using FEA, pre-processing, is constructing a finite element model of the structure to be analyzed. The input of a topological description of the structure's geometric features is required in most FEA packages. [3] This can be in either 1D, 2D, or 3D form, modeled by line, shape, or surface representation, respectively, although nowadays 3D models are predominantly used. The primary objective of the model is to realistically replicate the important parameters and features of the real model. [3] The simplest mechanism to achieve modeling similarity in structural analysis is to utilize pre-existing digital blueprints, design files, CAD models, and/or data by importing that into an FEA environment. Once the finite element geometric model has been created, a meshing procedure is used to define and break up the model into small elements. In general, a finite element model is defined by a mesh network, which is made up of the geometric arrangement of elements and nodes. Nodes represent points at which features such as displacements are calculated. FEA packages use node numbers to serve as an identification tool in viewing solutions in structures such as deflections. Elements are bounded by sets of nodes, and define localized mass and stiffness properties of the model. Elements are also defined

by mesh numbers, which allow references to be made to corresponding deflections or stresses at specific model locations.<sup>[3]</sup>

#### Analysis (computation of solution)

The next stage of the FEA process is analysis. The FEM conducts a series of computational procedures involving applied forces, and the properties of the elements which produce a model solution. Such a structural analysis allows the determination of effects such as deformations, strains, and stresses which are caused by applied structural loads such as force, pressure and gravity.

#### Post-processing (visualization)

These results can then be studied using visualization tools within the FEA environment to view and to fully identify implications of the analysis. Numerical and graphical tools allow the precise location of data such as stresses and deflections to be identified.

#### Applications of FEA to the mechanical engineering industry

A variety of specializations under the umbrella of the mechanical engineering discipline such as aeronautical, biomechanical, and automotive industries all commonly use integrated FEA in design and development of their products. Several modern FEA packages include specific components such as thermal, electromagnetic, fluid, and structural working environments. In a structural simulation FEA helps tremendously in producing stiffness and strength visualizations and also in minimizing weight, materials, and costs. FEA allows detailed visualization of where structures bend or twist, and indicates the distribution of stresses and displacements. FEA software provides a wide range of simulation options for controlling the complexity of both the modeling and the analysis of a system. Similarly, the desired level of accuracy required and the associated computational time requirements can be managed simultaneously to address most engineering applications. FEA allows entire designs to be constructed, refined, and optimized before the design is manufactured. This powerful design tool has significantly improved both the standard of engineering designs and the methodology of the design process in many industrial applications. [4] The introduction of FEA has substantially decreased the time taken to take products from concept to the production line.<sup>[4]</sup> It is primarily through improved initial prototype designs using FEA that testing and development have been accelerated.<sup>[5]</sup> In summary, the benefits of FEA include increased accuracy, enhanced design and better insight into critical design parameters, virtual prototyping, fewer hardware prototypes, a faster and less expensive design cycle, increased productivity, and increased revenue.[4]

#### Computer-aided design and finite element analysis in industry

The ability to model a structural system in 3D can provide a powerful and accurate analysis of almost any structure. 3D models, in general, can be produced using a range of common computer-aided design packages. Models have the tendency to range largely in both complexity and in file format, depending on 3D model creation software and the complexity of the model's geometry. FEA is a growing industry in product design, analysis, and development in engineering. The trend of utilizing FEA as an engineering tool is growing rapidly. The advancement in computer processing power, FEA, and modeling software has allowed the continued integration of FEA in the engineering fields of product design and development. In the past, there have been many issues restricting the performance and ultimately the acceptance and utilization of FEA in conjunction with CAD in the product design and

development stages. The gaps in compatibility between CAD file formats and FEA software limited the extent to which companies could easily design and test their products using the CAD and FEA combination, respectively. Typically, engineers would use specialist CAD and modeling software in the design of the product and then wish to export that design into a FEA package to test. However, those engineers who depended on data exchange through custom translators or exchange standards such as IGES or STEP cite occasional reliability problems causing unsuccessful exchange of geometry. <sup>[6]</sup> Thus, the creation of many models external to FEA environments was considered to be problematic in the success of FEA.

The current trend in FEA software & industry in engineering has been the increasing demand for integration between solid modeling and FEA analysis. During product design and development engineers require automatic updating of their latest models between CAD and FEA environments. There is still a need to improve the link between CAD and FEA, making them technically closer together. However, the demand for unitary CAD-FEA integration coupled with the improved computer and software developments has introduced a more robust and collaborative trend where compatibility problems are beginning to be eliminated. Designers are now beginning to introduce computer simulations capable of using pre-existing CAD files, without the need to modify and re-create models to suit FEA environments. [6]

One such CAD program with integrated FEA is SolidWorks from the SolidWorks Corporation, which is a midrange design tool offering an entry level FEA program called CosmosExpress. More advanced modules come in the shape of COSMOSMotion which is a more advanced SolidWorks module for simulating multi-body kinematic collisions and COSMOSWorks which handles more advanced linear static simulations. See also Pro/Engineer

## Current FEA trends in industry

#### Dynamic modeling

There is increasing demand for dynamic FEA modeling in the heavy vehicle industry. Many heavy vehicle companies are moving away from traditional static analysis and are employing dynamic simulation software. Dynamic simulation involves applying FEA in a more realistic sense to take into account the complicated effects of analyzing multiple components and assemblies with real properties.

#### Modeling assemblies

Dynamic simulation, used in conjunction with assembly modeling, introduces the need to fasten together components of different materials and geometries. Therefore, CAE tools should have comprehensive capabilities to easily and reliably model connectors, including joints that allow relative motion between components, rivets, and welds. Typical MSS models are composed of rigid bodies (wheels, axles, frame, engine, cab, and trailer) connected by idealized joints and force elements. Joints and links may be modeled as either rigid links, springs, or dampers in order to simulate the dynamic characteristics of real truck components. [7] Force transfer across assembly components through connectors makes them susceptible to high stresses. It is simpler and easier to idealize connectors as rigid links in these systems. This idealization provides a basic study of assembly behavior in terms of understanding system characteristics; engineers must model joining parameters like fasteners accurately when performing stress analysis to determine how failures might take place. [6] "Representing connectors as rigid links

assumes that connectors transfer loads across components without deforming and undergoing stress themselves. This unrealistic idealization yields incorrect predicted stresses in the regions local to the connectors, the exact locations where part failures will most likely initiate." [6] Understandably, the detailed inclusion of every connection point and/or mechanism in an assembly is impractical to model. [6] Therefore, improved representations of fasteners that are simple to use yet reliable should be investigated for use on a case-by-case basis. [6]

#### Current modeling techniques in industry

Engineers at various automotive companies currently model their vehicles using specialist dynamic FEA software. Each model contains a flexible body and chassis, springs, roll bars, axles, cab and engine suspension, the steering mechanism, and any frequency-dependent components such as rubber mounts. Extra details such as brakes and out-of-balance engine forces can be included on an "as-needed" basis. [8]

Dynamic FEA simulation enables a variety of maneuvers to be accurately tested. Tests such as steady-state cornering, roll-over testing, lane changing, J-turns, vibration analysis, collisions, and straight-line braking can all be conducted accurately using dynamic FEA. Non-linear and time-varying loads allow engineers to perform advanced realistic FEA, enabling them to locate critical operating conditions and determine performance characteristics.

As a result of the improved dynamic testing capabilities, engineers are able to determine the ultimate performance characteristics of the vehicle's design without having to take physical risks. As a result of dynamic FEA, the need for expensive destructive testing has been lessened substantially.<sup>[8]</sup>

## Leading products

- ALGOR
- ANSYS
- ABAOUS
- Femap
- LS-DYNA
- NASTRAN
- Sofistik

#### See also

- Finite element method in structural mechanics
- Finite element method
- Rayleigh-Ritz method
- Meshfree methods

#### References

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- Grube, Kris W., A Process for Investigating Geometric Sensitivity and Optimization of a Vehicle Structure, Ford Research Publication EM-89-1989.

#### External links

- ASME Eyewitness Series: Finite Element Method (FEM)
   (http://www.asme.org/Communities/History/Resources/Interactive\_Timeline.cfm) Historical Timeline
- "Early Masters of the Mesh" (http://www.memagazine.org/supparch/medesign/mesh/mesh.html) -Mechanical Engineering Magazine Online (ASME)
- Animations of Nonlinear Event Simulaton (http://www.nenastran.com/newnoran/animations)
- FEA in the Semiconductor Industry (http://www.siliconfareast.com/fea.htm)
- Finite Element Analysis Resources (http://www.feadomain.com/)
- FEAinformation (http://www.feainformation.com/) Industry leaders in software and hardware
- Finite Element Books (http://www.solid.ikp.liu.se/fe/index.html) Books bibliography
- Historical Outline of Matrix Structural Analysis: A Play in Three Acts (http://www.colorado.edu/engineering/CAS/Felippa.d/FelippaHome.d/Publications.d/Report.CU-CAS-00-13.pdf)
- Internet Finite Element Resources
  (http://homepage.usask.ca/~ijm451/finite/fe\_resources/fe\_resources.html) an annotated list of FEA links and programs
- Mesh Generation & Grid Generation on the Web (http://www-users.informatik.rwth-aachen.de/~roberts/meshgeneration.html) Software (http://www-users.informatik.rwth-aachen.de/~roberts/software.html) (public domain & commercial)
- Meshing Research Corner (http://www.andrew.cmu.edu/user/sowen/mesh.html) Software Survey (http://www.andrew.cmu.edu/user/sowen/softsurv.html)
- Mini-FEA (http://engineering-education.com/miniFEA/) Free web-based package that teaches students and other beginners how to "do FEA"
- NAFEMS (http://www.nafems.org/) International Association for the Engineering Analysis Community
- Z88 (http://www.z88.org/) OpenSource FEA program for Windows & LINUX/UNIX
- Finite Element Analysis news (http://engineeringbuzz.com/tags/Finite-Element-Analysis) Latest Finite Element Analysis news from across the web
- Magnitude (http://www.feamag.com/) Articles on both advanced and beginning FEA Topics.

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